On the kinematics of deep inelastic scattering in the covariant approach

Petr Zavada

Institute of Physics AS CR, Na Slovance 2, CZ-182 21 Prague 8, Czech Republic

Abstract. We study the kinematics of deep inelastic scattering corresponding to the rotationally symmetric distribution of quark momenta in the nucleon rest frame. It is shown that the rotational symmetry together with Lorentz invariance can impose constraints on the quark intrinsic momenta. Obtained constraints are discussed and compared with the available experimental data.

Keywords: deep inelastic scattering, covariant parton model, light cone, quark transversal momentum, nucleon rotational symmetry
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The motion of quarks inside the nucleons plays an important role in some effects which are at present intensively investigated both experimentally and theoretically. Actual goal of this effort is to obtain a more consistent 3-D picture of the quark-gluon structure of nucleons. For example the quark transversal momentum creates the asymmetries in particle production in polarized (SIDIS) or in unpolarized (Cahn effect) experiments on deep inelastic scattering (DIS). Relevant tool for the study of these effects is the set of the transverse momentum dependent distributions (TMDs). Apparently, a better understanding of the quark intrinsic motion is also a necessary condition to clarify the role of quark orbital angular momenta in generating nucleon spin.

We have paid attention to these topics in our recent studies, see [1, 2, 3, 4, 5] and citations therein. In particular we have shown that the requirements of Lorentz invariance (LI) and the nucleon rotational symmetry in its rest frame (RS), if applied in the framework of the 3-D covariant quark – parton model (QPM), generate a set of relations between parton distribution functions. Recently we obtained within this approach relations between usual parton distribution functions and the TMDs. The Wanzura-Wilczek approximate relation (WW) and some other known relations between the g_1 and g_2 structure functions were similarly obtained in the same model before [7]. Let us remark that the WW relation has been obtained independently also in another approaches [11, 10] in which the LI represents a basic input.

The aim of the present report is to consistently apply the assumption LI&RS to the kinematics of DIS and to obtain the constraints on related kinematical variables. That is a complementary task to the study of above mentioned relations between distribution functions, which depend on these variables. So, the report can be considered as an addendum to our former papers related to the covariant QPM [1, 2, 3, 4, 5, 6, 7, 8, 9]. However, general validity of obtained results may exceed the parton model framework.

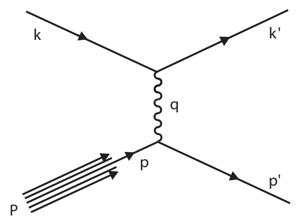

FIGURE 1. Diagram describing DIS as a one photon exchange between the charged lepton and quark. Lepton and quark momenta are denoted by k, p (k', p') in initial (final) state, P is initial nucleon momentum.

The Bjorken variable and light-cone coordinates

First, let us shortly remind the properties of the Bjorken variable

$$x_B = \frac{Q^2}{2Pq}, \tag{1}$$

which plays a crucial role in phenomenology of lepton – nucleon scattering. Regardless of mechanism of the process, this invariant parameter satisfies

$$0 \leq x_B \leq 1, \tag{2}$$

for complete proof of this general relation see [12]. Now let us consider QPM, where the process of lepton – nucleon scattering is initiated by the lepton interaction with a quark (see Fig. 1), which obeys

$$p' = p + q, \qquad p'^2 = p^2 + 2pq - Q^2; \qquad Q^2 = -q^2. \tag{3}$$

The second equality implies

$$Q^2 = 2pq - \delta m^2; \qquad \delta m^2 = p'^2 - p^2, \tag{4}$$

which with the use of relation (1) gives

$$\frac{pq}{Pq} = x_B \left(1 + \frac{\delta m^2}{Q^2}\right). \tag{5}$$

The basic input for the construction of QPM is the assumption

$$Q^2 \gg \delta m^2, \tag{6}$$

which allows us to identify

$$x_B = \frac{Q^2}{2Pq} = \frac{pq}{Pq} \tag{7}$$

and to directly relate the quark momentum to the parameters of scattered lepton. Moreover, if one assumes

$$Q^2 \gg 4M^2 x_B^2, \tag{8}$$

where M is the nucleon mass, then one can identify

$$x_B \simeq x \equiv \frac{p_0 - p_1}{P_0 - P_1} \tag{9}$$

in any reference frame in which direction of the first axis is defined by the vector $\mathbf{q}$ (see [12]). The last relation expressed in the nucleon rest frame reads

$$x = \frac{p_0 - p_1}{M}, \tag{10}$$

which after inserting into (2) gives

$$0 \leq \frac{p_0 - p_1}{M} \leq 1. \tag{11}$$

However the most important reason why we require large Q^2 is in physics. If we accept scenario when a probing photon interact with a quark, we need sufficiently large momentum transfer Q^2 at which the quarks can be considered as effectively free due to asymptotic freedom. At small Q^2 the picture of quarks (with their momenta and other quantum numbers) inside the nucleon disappear.

Rotational symmetry

The RS means that the probability distribution of the quark momenta $\mathbf{p} = (p_1, p_2, p_3)$ in the nucleon rest frame depends, apart from Q^2, on $|\mathbf{p}|$. It follows that also $-\mathbf{p}$ is allowed, so together with the inequality (11) we have

$$0 \leq \frac{p_0 + p_1}{M} \leq 1. \tag{12}$$

The combinations of (11),(12) imply

$$0 \leq |p_1| \leq p_0 \leq M, \qquad |p_1| \leq \frac{M}{2}. \tag{13}$$

And if we again refer to RS, then further inequalities are obtained:

$$0 \leq |p| \leq p_0 \leq M, \quad |p| \leq \frac{M}{2}, \quad 0 \leq p_T \leq p_0 \leq M, \quad p_T \leq \frac{M}{2}, \tag{14}$$

where

$$|p| = \sqrt{p_1^2 + p_2^2 + p_3^2}, \qquad p_T = \sqrt{p_2^2 + p_3^2}.$$

Apparently, the above inequalities are valid also for average values $\langle p_0 \rangle, \langle p_1 \rangle, \langle |p| \rangle$ and $\langle p_T \rangle$. In addition, if one assumes that p_T- distribution is a decreasing function, then necessarily

$$\langle p_T \rangle \leq \frac{M}{4}. \tag{15}$$

The above relations are valid for sufficiently high Q^2 suggested by Eqs. (6) and (8). Let us note that the on-mass-shell assumption has not been applied for obtaining these relations. On the other hand the additional on-mass-shell condition $m^2 = p^2 = p_0^2 - \mathbf{p}^2$ allows us to obtain [9] the more strict relations like

$$x \geq \frac{m^2}{M^2}, \qquad p_0 \leq \frac{M^2+m^2}{2M}, \qquad |\mathbf{p}| \leq \frac{M^2-m^2}{2M}, \tag{16}$$

or

$$p_T^2 \leq M^2 \left(x - \frac{m^2}{M^2} \right) (1-x). \tag{17}$$

Of course, it is clear that in general the on-mass-shell assumption is not realistic.

Now let us make a few comments on the relations obtained in the previous part:

i) The ratio x of light-cone variables (9) has a simple interpretation in the frame, where the proton momentum is large - x is the fraction of this momentum carried by the quark. On the other hand interpretation of the same variable in the nucleon rest frame is more complicated. In this frame the quark transversal momentum cannot be neglected and x depends on the both, longitudinal and transversal quark momenta components. In the limit of massless quarks the connection between the variable x in (10) and the quark momenta components is given by the relations:

$$x = \frac{p_0 - p_1}{M}; \quad p_0 = \sqrt{p_1^2 + p_T^2}, \quad p_1 = -\frac{Mx}{2}\left(1 - \frac{p_T^2}{M^2x^2}\right), \quad p_0 = \frac{Mx}{2}\left(1 + \frac{p_T^2}{M^2x^2}\right). \tag{18}$$

The sense and usefulness of these variables have been demonstrated in our recent papers on TMDs [1, 2]. So, the value of invariant variable x does not depend on the reference frame, but its interpretation e.g. in the rest frame differs from that in the infinite momentum frame.

ii) The relations (14), (15), which follow from RS, can be confronted with the experimental data on $\langle p_T \rangle$ or $\langle \mathbf{p} \rangle$. We have discussed the available data in [1, 2] and apparently our relations would prefer the set of lower values $\langle p_T \rangle$ corresponding to the 'leptonic data'. On the other hand the second set giving substantially greater $\langle p_T \rangle$ and denoted as the 'hadronic data', seems to contradict these relations. If we believe this data, then the contradiction means that some assumption, from which the relations follow, is false. Here, probably the most critical can be inequality (6). But let us notice, failure of condition (6) would also imply that the Bjorken variable cannot be replaced by the light-cone ratio, Eq. (7). And then correspondingly the light-cone formalism would be called into question. In this way, the large intrinsic quark momenta ($p_T > M/2$) are incompatible with the light-cone formalism combined with the RS. No such problem arises provided the data satisfy relations (14), (15). Apparently a further study is needed to clarify these questions.

iii) The LI&RS are basic inputs for our covariant QPM. In [2] we have suggested that the RS, if applied on the level of QPM, follows from the covariant description. The point is as follows. The construction of covariant parton models starts from probability distribution of the quark momenta. Covariant formulation requires that dependence on the quark momentum p can be only via an invariant term depending on p. In fact the parton models allow only one such term: pP. This input is applied not only in our QPM, but also in another approaches, see e.g. [10, 11, 13]. The invariant term pP expressed in

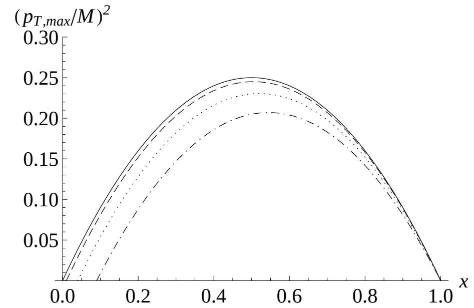

FIGURE 2. Upper limit of the quark transversal momentum as a function of x for $\mu = 0$ (solid line), 0.1 (dashed line), 0.2 (dotted line) and 0.3 (dash-dotted line).

the nucleon rest frame equals $p_0 M$, which is equivalent to the RS. Another theoretical reasons for RS have been discussed in [8]. In our QPM we usually assume $m \to 0$, or more exactly $0 < m \ll p_0 \leq M$. Apart from the relations (17) we have shown in [9] that related distribution function has a maximum at $x = m/M$. Since existing data [14] cover region $x \gtrsim 0.00006$ where no maximum is observed, it follows that the model does not contradict the data provided $m/M \lesssim 0.00006$. That is why we assume $m \to 0$ in our QPM. At the same time we realize the notion of the quark mass is more complicated and exceeds the framework of QPM. For massless quarks the relation (17) implies

$$\left\langle p_T^2(x) \right\rangle \leq M^2 x(1-x). \tag{19}$$

One can check that the predictions given in [10, 11] on $\left\langle p_T^2(x) \right\rangle$ satisfy this inequality, which also means that $\langle p_T \rangle \to 0$ for $x \to 0$ or $x \to 1$.

iv) The relation (17) is obtained for the quarks on-mass-shell. In a more general case, where only inequalities (14) hold, this relation is replaced by

$$p_T^2 \leq M^2 \left(x - \frac{\mu^2}{M^2} \right)(1-x); \qquad \mu^2 \equiv p_0^2 - \mathbf{p}^2, \tag{20}$$

where the term μ^2 is not a fixed parameter corresponding to the mass, but only a number varying in the limits defined by (14). The last relation implies for any μ^2:

$$p_T^2 \leq M^2 x(1-x), \tag{21}$$

which is equivalent to the on-mass-shell relation (17) for $m = 0$. This general upper limit for p_T^2 depending on x is demonstrated in Fig. 2.

The above comments concern general relations (7) and (13)–(15), which in the region of sufficiently high Q^2 follow from the assumption LI&RS. Only the comment *iii)* is connected with the relations valid within specific model.

To conclude, in the present report we studied the kinematic constraints due to the rotational symmetry of the quark momenta distribution inside the nucleon. In particular, we have shown that the light-cone formalism combined with the assumption on the rotational symmetry in the nucleon rest frame allows only $p_T \leq M/2$. Only part of existing experimental data on $\langle p_T \rangle$ satisfies this bounds, but the another part does not. In general, the reconstruction of $\langle p_T \rangle$ from the DIS data is a model-dependent procedure. These are reasons why a more study is needed to clarify this issue.

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